What to Expect on the Placement Exam

Placement into:
MTH 122 or MTH 144

The ACCUPLACER placement exam is an adaptive test created by the College Board Educational Testing Service. This document was created to give prospective CLC student and other interested parties an overview of how the placement exam works and the types of questions one might expect to see on it.

General Information:
- The exam is computerized and is not timed.
- Questions are presented in a multiple-choice format. No partial credit is awarded.
- Scratch paper is provided.
- Handheld calculators are not allowed. The testing software provides a pop-up calculator for use on some questions.
- There is a limit to the number of times a student may take the placement exam.

CLC Mathematics department wants each student to do as well as possible on the placement exam and makes the following recommendations:
- Prepare for the exam. Don’t take it cold. Studying for the exam may decrease the number of math classes you are required to take. Free practice questions/answers are available online at the Accuplacer website, located at https://accuplacer.collegeboard.org/students. Other review materials are available at the Math Center. (847) 543-2449
- Take the exam when you are rested and refreshed.
- Allow plenty of time for testing so that you can relax and fully concentrate on what you are doing.
- Triple-check answers before moving on to the next question. The computer only knows if your answer is right or wrong. It cannot tell the difference between a careless mistake, a minor error, or a major error.
- Stay calm if you don’t know the answer to a question. Remember that the placement test is adaptive and is used to test many levels of mathematics. It increases or decreases the level of the questions based on your previous answers.

A student desiring to place into College Algebra (MTH 122) or Precalculus (MTH 145) must first pass the twelve question Basic Algebra portion of the exam. The student will then be given the opportunity to answer questions from the College Mathematics portion of the exam. Any student intending to place into College Algebra or Precalculus via the Math Placement Exam must be proficient with the types of algebra exercises illustrated as examples for entry into Intermediate Algebra (MTH 108). In addition, the student must be proficient with the types of exercises illustrated in this document.

NOTE: The examples provided below are not intended to be a complete list of problem types. The examples are simply illustrations of problem types.
1. The student must be completely proficient with all aspects of linear equations in two variables, lines, and linear functions. The student must be able to go from the graph to the equation and vice versa.

Examples:

a) Graph the following: \( y = 3x - 7 \); \( 2y - 3x = 8 \); \( \frac{2}{3} x + \frac{3}{4} y = \frac{9}{2} \); \( \frac{x}{5} + \frac{y}{9} = 1 \)

b) What value must “a” take on for the line defined by \( y = ax - 7 \) to be parallel to the line defined by \( 2y - 3x = 8 \)?

c) What value must “a” take on for the line defined \( y = ax - 7 \) to be perpendicular to the line defined by \( 2y - 3x = 8 \)?

d) Write the equations of the following lines.

2. The student must be proficient with all properties of real valued exponents and be able to simplify expressions involving them.

Examples: Simplify the following:

a) \( \frac{x^4 y^5 z^{\sqrt{3}}}{y^{-7} z^0 x^5} \)

b) \( \frac{q^{-3a} r^b q^a r^{-3b}}{r^b q^2 r} \)

c) \( \left( \frac{q^{-2} r^3 z^2}{r^3 q z^3} \right)^2 \)

3. The student must be able to recognize the equation of and the graph of simple quadratic functions along with horizontal and vertical translations of them.

Examples:

a) Graph the following:

\( y = x^2 + 5 \) \( y = -\frac{1}{2} x^2 - 2 \) \( y = (x - 2)^2 + 5 \) \( y = -(x + 2)^2 + 1 \)

b) Which equation below could correspond to the following graph?

\[ i) \ y = x^2 + 5 \quad ii) \ y = -\frac{1}{2} x^2 - 2 \]

\[ iii) \ y = (x - 2)^2 + 5 \quad iv) \ y = -(x + 2)^2 + 1 \]
4. The student must be able to solve simple linear and non-linear systems of equations.

**Examples:** Solve the following systems of equations.

\[
\begin{align*}
\text{a) } & \quad y = 3x - 8 & \quad \text{b) } & \quad 16x - 2y = 2 & \quad \text{c) } & \quad x + y - z = 2 \\
\quad & \quad y = 3x^2 - 17 & \quad & \quad y - 2x^2 - 7 = 0 & \quad & \quad 2x + 3z = -3 \\
\text{ } & \quad \text{ } & \quad & \quad \text{ } & \quad & \quad y = 14
\end{align*}
\]

5. The student must be able to solve simple linear and non-linear equations, which involve fractions.

**Examples:** Solve the following equations.

\[
\begin{align*}
\text{a) } & \quad \frac{3x}{2(x-1)} + \frac{5}{x-1} = \frac{-1}{2} & \quad \text{b) } & \quad \frac{x}{x-1} - \frac{2}{x+1} = \frac{4}{x^2 - 1} \\
\text{c) } & \quad \frac{2x}{3} + \frac{5x}{2} = \frac{7x}{6} & \quad \text{d) } & \quad \frac{ax}{b} - bx = \frac{c}{a^2}
\end{align*}
\]

6. The student must be able to perform composition of functions.

**Example:** Given: \( f(x) = 3x^2 - 2 \) and \( g(x) = -2x \)

a) What is \( f(g(x)) \)?  
   b) What is \( g(f(x)) \)?  
   c) What is \( f(-x) \)?  
   d) What is \( g(g(x)) \)?

7. The student needs to be able to solve quadratic equations using factoring, the zero factor technique, and the quadratic formula.

**Examples:** Solve the following.

\[
\begin{align*}
\text{a) } & \quad x^2 - x - 12 = 0 & \quad \text{b) } & \quad 6x^2 - x - 15 = 0 & \quad \text{c) } & \quad x^2 - 6x + 7 = 0
\end{align*}
\]

8. The student must be able to add and subtract moderately difficult algebraic fractions using least common denominators.

**Examples:**

\[
\begin{align*}
\text{a) } & \quad \frac{2x - 7}{x^2 - x - 12} - \frac{2x - 3}{x^2 + 6x + 9} & \quad \text{b) } & \quad \frac{z}{z^2 - 4} + \frac{3z + 5}{4z - 8} - \frac{3}{z} & \quad \text{c) } & \quad \frac{x + 5}{4x^2 - 9} - \frac{x - 1}{2x^2 - x - 3}
\end{align*}
\]

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9. The student must be able to multiply and expand polynomials.

**Examples:** Perform the following multiplications

a) \((3x - 7y)(2x + 3y)(6x - 8)\) 

b) \((3x^2y + 5z^3)^3\)

c) \((3x - 2y + 5z - 4w)^2\)

10. The student must be able to factor common factors from a variety of algebraic expressions.

**Examples:** Recognize the Greatest Common Factor (GCF) and factor the expression.

a) \(a^3 + bax^2 + cxa\)

b) \(x^2yzp + x^2zyq - 3x^2y\)

c) \((3x - 2)(2p - 7) + (42x^2 - 3q)(2p - 7) - (2p - 7)\)

11. The student must be proficient with factoring algebraic expressions.

**Examples:** Completely factor the following.

a) \(x^2 + 7x + 12\) 

b) \(25z^2 - 64q^2\) 

c) \(51x^2 + 8x - 3\)

d) \((2x - 2)(3x - 7) + (2x - 2)(2x + 7)\)

12. The student must be proficient with simplifying all types of rational algebraic expressions including complex algebraic fractions.

**Examples:** Simplify the following algebraic expressions.

a) \(\frac{x^2 - 5x - 14}{x^2 - 8x + 7}\)

b) \(\frac{16 - x^4}{x^3 - 2x^2 + 4x - 8}\)

c) \(\frac{5x^2 + 19x - 4}{x^2 - 4} \div (5x - 1)\)

d) \(\frac{1}{1 + x}\)

e) \(\frac{a + b}{a - b}\)

f) \(\frac{1}{a^2 + 2ab + b^2}\)

13. The student must be proficient with simple absolute value expressions, equations and inequalities.

**Examples:**

a) Solve for \(x\): \(|x + 1| = 6\)

b) Solve for \(x\): \(|2x - 3| \leq 7\)

c) \(|x| + 2x = ?\) if \(x < 0\)

d) \(|x| + x = ?\) if \(x > 0\)
14. The student must be proficient with the basics of exponential and logarithmic functions.

**Examples:**

a) If \( \log_5 2 = x \), then

\[
\begin{align*}
\text{i)} & \quad 5 = x \\
\text{ii)} & \quad 2 = 5^x \\
\text{iii)} & \quad 5 = x^2 \\
\text{iv)} & \quad x = 25
\end{align*}
\]

b) The graph shown could be the graph of

\[
\begin{align*}
\text{i)} & \quad y = \log_2 x \\
\text{ii)} & \quad x = \log_5 2 \\
\text{iii)} & \quad y = 2^x \\
\text{iv)} & \quad y = x^2
\end{align*}
\]

c) The inverse function corresponding to the function \( y = \log_b x \) is given by

\[
\begin{align*}
\text{i)} & \quad y = b^x \\
\text{ii)} & \quad x = b^y \\
\text{iii)} & \quad b = \log_y x \\
\text{iv)} & \quad y = \log_b x
\end{align*}
\]
What to Expect on the Placement Exam

Placement into:
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Solutions

1. The student must be completely proficient with all aspects of linear equations in two variables, lines, and linear functions. The student must be able to go from the graph to the equation and vice versa.

Examples:

a) Graph the following:
   \[ y = 3x - 7; \quad 2y - 3x = 8; \quad \frac{2}{3}x + \frac{3}{4}y = \frac{9}{2}; \quad \frac{x}{5} + \frac{y}{9} = 1 \]

   **Answers:**

   ![Graphs of linear equations]

b) What value must “a” take on for the line defined by \( y = ax - 7 \) to be parallel to the line defined by \( 2y - 3x = 8 \)?

   **Answer:** 2\( y = 3x + 8 \), so \( y = \frac{3}{2}x + 4 \). \( a = 3/2 \)

c) What value must “a” take on for the line defined \( y = ax - 7 \) to be perpendicular to the line defined by \( 2y - 3x = 8 \)?

   **Answer:** 2\( y = 3x + 8 \), so \( y = \frac{3}{2}x + 4 \). \( a = -2/3 \)
d) Write the equation of the following lines.

**Answers:**

\[ y = \frac{4}{3} x + 4 \]  
\[ y = \frac{3}{2} x \]

2. The student must be proficient with all properties of real valued exponents and be able to simplify expressions involving them.

**Examples:**

\[ x^4 y^{5\sqrt{3}} \]
\[ = \frac{y^{12\sqrt{3}}}{x} \]

\[ \frac{q^{-3a} r^b q^a r^{-3b}}{r^{-b} q^2 r} \]
\[ = q^{-2a-2} r^{-b-1} = \frac{1}{q^{2a+2} r^{b+1}} \]

\[ \left( \frac{q^{-2} r^3 z^2}{r^{-3} q^z} \right)^{\frac{1}{2}} \]
\[ = (q^{-3} r^6 z^{-1})^{\frac{1}{2}} = q^6 r^{-12} z^{\frac{1}{2}} = \frac{q^6 z^2}{r^{12}} \]

3. The student must be able to recognize the equation of and the graph of simple quadratic functions along with horizontal and vertical translations of them.

**Examples:**

a) Graph the following:

\[ y = x^2 + 5 \]
\[ y = -\frac{1}{2} x^2 - 2 \]
\[ y = (x - 2)^2 + 5 \]
\[ y = -(x + 2)^2 + 1 \]
b) Which equation below could correspond to the following graph?

Answer:

- i) \( y = x^2 + 5 \)
- ii) \( y = -\frac{1}{2} x^2 - 2 \)
- iii) \( y = (x - 2)^2 + 5 \)
- iv) \( y = -(x + 2)^2 + 1 \)

4. The student must be able to solve simple linear and non-linear systems of equations.

**Examples:** Solve the following systems of equations. **Answers:**

<table>
<thead>
<tr>
<th>( x + y - z = 2 )</th>
<th>( 2x + 3z = -3 )</th>
<th>( y = 14 )</th>
<th>( x + 14 - z = 2 )</th>
<th>( x - z = -12 )</th>
<th>( 2x + 3z = -3 )</th>
<th>( 3x - 3z = -36 )</th>
<th>( 2x + 3z = -3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x = -39 )</td>
<td>( x = \frac{-39}{5} )</td>
<td>Solution: ( \left( -\frac{39}{5}, 14, \frac{21}{5} \right) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. The student must be able to solve simple linear and non-linear equations, which involve fractions.

**Examples:**

<table>
<thead>
<tr>
<th>a) $\frac{3x}{2(x-1)} + \frac{5}{x-1} = \frac{-1}{2}$</th>
<th>$3x + 10 = -(x-1)$</th>
<th>$x = \frac{9}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x = -9$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| b) $\frac{x}{x-1} - \frac{2}{x+1} = \frac{4}{x^2-1}$ | $x(x+1) - 2(x-1) = 4$ | $x = 2$ (x \neq 1$ because it would cause division by 0 in the original problem)
| $x^2 + x - 2x + 2 = 4$                           | $x^2 - x - 2 = 0$ |                  |
| $x - 2 = 0$                                      | $(x - 2)(x + 1) = 0$ |                  |
| c) $6\left(\frac{2}{3}x + \frac{5}{2}x\right) = \frac{-7x}{6} \cdot 6$ | $4x + 15x = -7x$ | $26x = 0$ |
| $6\left(\frac{2}{3}x + \frac{5}{2}\right) = -7x$ | $x = 0$           |                  |
| d) $\frac{ax}{b} - bx = \frac{c}{a^2}$          | $a^3x - a^2b^2x = bc$ | $x = \frac{bc}{a^3-a^2b^2}$ |
| $x = \frac{bc}{a^3-a^2b^2}$                     | $bc$             | $= \frac{bc}{a^2(a-b^2)}$ |

6. The student must be able to perform composition of functions.

Given: $f(x) = 3x^2 - 2$ and $g(x) = -2x$

**Examples:**

<table>
<thead>
<tr>
<th>a) What is $f(g(x))$?</th>
<th>$f(g(x)) = 3(-2x)^2 - 2 = 12x^2 - 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) What is $g(f(x))$?</td>
<td>$g(f(x)) = -2(3x^2 - 2) = -6x^2 + 4$</td>
</tr>
<tr>
<td>c) What is $f(-x)$?</td>
<td>$f(-x) = 3(-x)^2 - 2 = 3x^2 - 2$</td>
</tr>
<tr>
<td>d) What is $g(x)$?</td>
<td>$g(x) = -2(-2x) = 4x$</td>
</tr>
</tbody>
</table>

7. The student needs to be able to solve quadratic equations via factoring and the zero factor technique and also via the quadratic formula.

**Examples:** Solve the following.

<table>
<thead>
<tr>
<th>a) $x^2 - x - 12 = 0$</th>
<th>$(x - 4)(x + 3) = 0$, so $x = 4$ or $x = -3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) $6x^2 - x - 15 = 0$</td>
<td>$(3x - 5)(2x + 3) = 0$, so $x = \frac{5}{3}$ or $x = -\frac{3}{2}$</td>
</tr>
<tr>
<td>c) $x^2 - 6x + 7 = 0$</td>
<td>$x = \frac{6 \pm \sqrt{6^2 - 4(1)(7)}}{2(1)} = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$</td>
</tr>
</tbody>
</table>
8. The student must be proficient with and be able to add and subtract moderately difficult algebraic fractions using least common denominators.

**Examples:**

**Answers:**

a) \[ \frac{2x - 7}{x^2 - x - 12} - \frac{2x - 3}{x^2 + 6x + 9} = \frac{10x - 33}{(x - 4)(x + 3)^2} \]

b) \[ \frac{z}{z^2 - 4} + \frac{3z + 5 - 3}{4z - 8} = \frac{3z^3 + 3z^2 + 10z + 48}{4z(z - 2)(z + 2)} \]

c) \[ \frac{x + 5}{4x^2 - 9} - \frac{x - 1}{2x^2 - x - 3} = \frac{-x^2 + 5x + 8}{(x + 1)(2x - 3)(2x + 3)} \]

9. The student must be able to multiply and expand polynomials.

**Examples:**

**Answers:**

a) \[(3x - 7 y)(2x + 3 y)(6x - 8) = 36x^3 - 30x^2 y - 48x^2 - 126xy^2 + 40xy + 168y^2 \]

b) \[(3x^2 y + 5z^3)^2 = 9x^4 y^2 + 30x^2 yz^3 + 25z^6 \]

c) \[(3x - 2 y + 5z - 4w)^2 = 16w^2 - 24wx + 16wy - 40wz + 9x^2 - 12xy + 30xz + 4y^2 - 20yz + 25z^2 \]

10. The student must be able to factor common factors from a variety of algebraic expressions.

**Examples:**

**Answers:**

a) \[ ax^3 + bazx^2 + cxa = ax^4 (x^2 + bzx + c) \]

b) \[ x^2 yz p + x^2 zy q - 3 x^2 y = x^2 y(zp + zq - 3) \]

c) \[ (3x - 2)(2p - 7) + (42x^2 - 3q)(2p - 7) - (2p - 7) = (2p - 7)(3x - 2 + 42x^2 - 3q - 1) = (2p - 7)(42x^2 + 3x - 3q - 3) = 3(2p - 7)(14x^2 + x - q - 1) \]

11. The student must be proficient with factoring algebraic expressions.

**Examples:**

**Answers:**

a) \[ x^2 + 7x + 12 = (x + 3)(x + 4) \]

b) \[ 25z^2 - 64q^2 = (5z - 8q)(5z + 8q) \]

c) \[ 51x^2 + 8x - 3 = (3x + 1)(17x - 3) \]

d) \[ (2x - 2)(3x - 7) + (2x - 2)(2x + 7) = 2(x - 1)(5x) = 10x(x - 1) \]
12. The student must be proficient with simplifying all types of rational algebraic expressions including complex algebraic fractions.

**Examples:**

a) \[ \frac{x^2 - 5x - 14}{x^2 - 8x + 7} \]

\[ = \frac{(x-7)(x+2)}{(x-7)(x-1)} = \frac{x+2}{x-1} \]

b) \[ \frac{16 - x^4}{x^3 - 2x^2 + 4x - 8} \]

\[ = \frac{(2-x)(2+x)(x^2+4)}{(x-2)(x^2+4)} = -(x+2) \]

c) \[ \frac{5x^2 + 19x - 4}{x^2 - 4} \div (5x - 1) \]

\[ = \frac{(5x-1)(x+4)}{(x^2-4)(5x-1)} = \frac{x+4}{x^2-4} \]

d) \[ \frac{1}{1+x} \div \frac{1}{1-x} + 1 \]

\[ = \frac{1}{1+x} \cdot \frac{1-x}{1-x} = \frac{1}{(x+1)}(1-x) = \frac{x-1}{(x-2)(x+1)} \]

e) \[ \frac{a+b}{a - b} \div \frac{a^2 + 2ab + b^2}{a^2 - b^2} \]

\[ = \frac{(a+b)(a-b)}{(a+b)(a+b)} = \frac{a+b}{a-b} = 1 \]

f) \[ \frac{1}{a^2} + \frac{2}{ab} \div \frac{3}{ab^2} + \frac{1}{b} \]

\[ = \frac{b+2a}{a^2b} \div \left( \frac{ab^2}{3+ab} \right) = \frac{b(2a+b)}{a(ab+3)} \]

13. The student must be proficient with simple absolute value expressions, equations and inequalities.

**Examples:**

a) Solve for \( x \): \( |x + 1| = 6 \)

\( x + 1 = 6 \) or \( x + 1 = -6 \), so \( x = 5 \) or \( x = -7 \)

b) Solve for \( x \): \( |2x - 3| \leq 7 \)

\( -7 \leq 2x - 3 \leq 7 \), so \( -4 \leq 2x \leq 10 \) and \(-2 \leq x \leq 5 \)

c) \( |x| + 2x = ? \) if \( x < 0 \)

if \( x < 0 \), \( |x| = -x \), so \(-x + 2x = x \)

d) \( |x| + x = ? \) if \( x > 0 \)

if \( x > 0 \), \( |x| = x \), so \( x + x = 2x \)
14. The student must be proficient with the basics of exponential and logarithmic functions.

Examples:

a) If \( \log_5 2 = x \), then

- i) \( 5 = x \)
- ii) \( 2 = 5^x \)
- iii) \( 5 = x^2 \)
- iv) \( x = 25 \)

b) The graph shown could be the graph of

- i) \( y = \log_2 x \)
- ii) \( x = \log_5 2 \)
- iii) \( y = 2^x \)
- iv) \( y = x^2 \)

c) The inverse function corresponding to the function \( y = \log_b x \) is given by

- i) \( y = b^x \)
- ii) \( x = b^y \)
- iii) \( b = \log_y x \)
- iv) \( y = \log_x b \)